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DISCUSSION

PROPAGATION OF NORMAL MODES DUE TO IMPULSIVE LOADING TO 3-D MEDIUM ON A RIGID BASEMENT[1]

DISCUSSION BY EDUARDO KAUSEL

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In a recent paper, Touhei[1] presented a supposedly new numerical method for the analysis of wave propagation in a layered medium, which he applied to study the motion elicited by a seismic pulse in a homogeneous stratum. As it turns out, there is an extensive literature on this method, which has been widely used for the past 25 years. It is now commonly referred to as the Thin Layer Method (TLM), and has found diverse applications in engineering and seismology. For example, our code PUNCH implementing this technique—developed by us some 16 years ago—is now being used in numerous ways around the world as an efficient tool to obtain the dynamic response of laminated media, including plates, strata of finite depth, and even layered media over elastic half-spaces in two or three dimensions. Among the most frequent applications of this method are in non-destructive testing of pavements, wave propagation in laminated plates and soils (including anisotropic or poroelastic media), soil–structure and fluid–structure interaction, and in seismic simulation.

The method was first used in 1970 by Lysmer[2] to study the propagation of seismic Rayleigh waves in layered strata; he also formulated an equation based on Rayleigh's quotient to obtain the group velocities and dispersion characteristics of the waves as a direct by-product of the calculation. Like Touhei, he observed that some modes had a negative group velocity, and he gave a thorough interpretation for this phenomenon. Two years later, Waas[3] developed a very substantial extension to Lysmer's method in which he obtained the characteristic equations directly from a variational formulation, and not as a limiting process to a finite element mesh, as Lysmer had done. He applied the method extensively to problems in two dimensions (SV-P and SH waves) as well as for torsional waves. Moreover, he developed a very efficient algorithm for extracting the normal modes, studied in detail the characteristics of waves in a homogeneous stratum, and applied the method to obtain a super-accurate transmitting boundary for finite element representations of irregular two-dimensional soil media.[3,4] (These so-called consistent boundaries are equivalent to a virtual continuation of the finite element mesh to infinity; they have been implemented in well-known computer codes such as FLUSH). At about this time, the method was also used by Dong and Nelson[5,6] to study the vibrations of laminated orthotropic plates, and so did also Srinivas[7] using a closely related method.

In 1974, we extended the works of Lysmer and Waas, obtaining the complete formulation for three-dimensional loads in layered media exhibiting cylindrical material symmetry;[8] we also developed a consistent boundary for cylindrical co-ordinates, found that the associated eigenvalue problems for the cylindrical case was identical to a combination of the eigenvalue problems for the two plane-strain cases, and that this eigenvalue problem was independent of the azimuthal Fourier number. We then applied the method to study the dynamic and seismic response of building foundations.[9,10] In 1977, we also presented a methodology based on the TLM to study layered media of finite width, such as plates with arbitrary edge conditions.[11]

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Perhaps the most fundamental advance to the TLM came in the early 1980s when Tajimi,[12] Waas,[13] and Kausel[14] independently applied the method to obtain the response of a layered system of finite depth to concentrated sources acting within (or on) the medium. Of these three, the most well-known came to be the latter, since it provided the most general framework for handling loads with arbitrary spatial-temporal characteristics via Fourier and Hankel transforms, and included detailed expressions for the consistent strains and stresses. Based on this work, Kausel and Peek[15] presented the Green's functions for point sources of various types—including a seismic double couple—which now lie at the heart of numerous programs and procedures for the analysis of wave motion in layered media (e.g. PUNCH, SASSI, SASW). A good number of other applications and extension by us followed since then, including the capability to assess the response of infinite media, such as a layer over an elastic half-space[16–19] and the formulation for materials with arbitrary anisotropy in both the frequency domain[20] and in the time domain.[21,22] We have also used this method to study dynamically loaded structures embedded in layered media, such as tunnels, by means of the Boundary Element Method.[23]

Over the past 15 years, the TLM has been applied for diverse purposes. Tassoulas[24,25] developed sophisticated finite elements of arbitrary width for the analysis of plane and cylindrical problems, and applied the method to obtain the dynamic impedances of cylindrical and annular footings embedded in layered media.[26] Zinn[27] formulated the method for concentric cylindrical layers, and studied the propagation of waves in cylindrically laminated rods. More recently, applications have also included coupled soil–fluid–structure interaction,[28–32] as well as poroelastic media.[33] The reader is referred to the additional bibliography[34–42] for further works related to this method.

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